A Gap in the Quarkyonic matter

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It has recently been suggested that at a reasonably large chemical potential a confining quarkyonic matter is formed that consists of the quark Fermi sea and confined hadrons on top of this Fermi sea. We study some properties of this matter. It is demonstrated that below the chiral restoration point there are gapless excitations of this matter through excitations of the Goldstone bosons. Above the chiral restoration point the single quarks are still removed from the spectrum of excitations and the only possible excitations are confined color-singlet hadrons with finite mass. Hence there appears a gap in the excitation spectrum of the quarkyonic matter that should be crucially important for its properties above the chiral restoration point. This gap is of a new type and is not related with the condensation of the fermionic system into a quasibosonic system. It is only due to such properties as confinement and manifest chiral symmetry at the same time.

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1. Introduction. Very recently McLerran and Pisarski have suggested the unusual structure of the QCD phase diagram in the large N_c limit [1]. Previously it was considered natural that at a reasonably large density above the critical one and not so large temperature the QCD matter is a deconfined system of quarks and gluons, see Fig. 1. In such a situation, however, the pressure should be proportional to N_c^2, because it is dominated by the deconfined gluons in the adjoint representation of $SU(N_c)$. At the very large densities the perturbative calculation of the pressure is adequate and it produces a pressure that scales like N_c. A resolution of this paradox is that the system at the reasonably large densities and low temperatures represents essentially a Fermi sea of valence quarks (baryons are in a strong overlap and hence it is impossible to decide to which particular baryon a given quark belongs) and confined hadrons on top of this sea, see Fig. 2. Then the quarks in the Fermi sea produce a pressure $\sim N_c$ while confined hadrons at the Fermi surface contribute as ~ 1. There cannot be contribution to the pressure from the deconfined gluons and consequently the system is manifestly confined. Such a matter was termed as "quarkyonic".

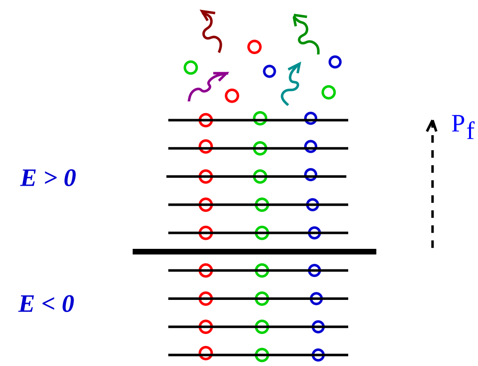

FIG. 1: A schematic cartoon of the deconfined quark-gluon matter at a finite chemical potential.

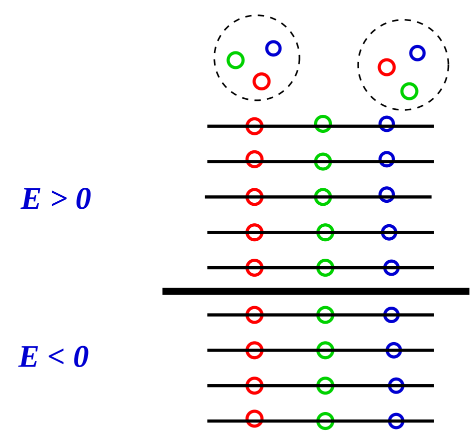

FIG. 2: A schematic cartoon of the quarkyonic matter at a finite chemical potential.

At the critical chemical potential there should be a chiral restoration phase transition in this quarkyonic matter. Then on top of the Fermi sea of quarks there must exist confined but chirally symmetric hadrons. This contradicts to our naive intuition and to simple models of confinement and chiral symmetry breaking. It has been demonstrated, however, that it is not so and actually it is quite natural to expect confined but chirally symmetric hadrons on top of the Fermi sea above the chiral restoration point [2]. A possible mechanism for such hadrons has also been clarified.

Here we would like to address some general and crucially important properties of the quarkyonic matter below and above the chiral restoration point, that have not been discussed in the papers above. In the chirally broken phase there always exists a gapless excitation of the quarkyonic matter, that is due to excitations of the Goldstone bosons on top of the Fermi sea. Above the chiral restoration point, however, there is no a gapless excitation of the quarkyonic matter. The gap is given by the energy of lowest lying pion-like excitation, that has a *finite* energy which depends on the chemical potential. Then the properties of the quarkyonic matter are essentially determined by the presence of the gap. This is quite an unusual situation, because typically a gap in the excitation spectrum of the Fermi system is attributed to the condensation of the fermion pairs due to some pairing interaction between fermions that leads to the quasi bosonic system with such properties like superconductivity or superfluidity [3, 4, 5]. In our case, however, a presence of the gap at the chemical potential above the critical one is not due to condensation of the Cooper pairs. It has a completely different nature related to such simultaneous properties of the system as confinement and manifest chiral symmetry. We are not aware of any example of this kind in the usual Fermi systems and consequently the quarkyonic matter above the chiral restoration point would represent a Fermi system with new and yet unknown properties.

2. Quarkyonic matter below and above the chiral restoration point. The property which we discuss, namely the absence of a gap in the quarkyonic matter below the critical chemical potential and its presence above the critical chemical potential is a very general one and is actually related only to confinement and existence/nonexistence of the massless Goldstone excitations at different chemical potentials. To see it explicitly and to understand microscopical reasons one needs a model that is manifestly chirally symmetric, confining and provides spontaneous breaking of chiral symmetry. Such a model might not be exactly equivalent to QCD. However, if it incorporates these properties it provides an insight and illustrates a general QCD property of the quarkyonic matter in the large N_c limit.

In this context we use the only known exactly solvable confining and chirally symmetric model in four dimensions that is a generalization of the 1+1 dimensional 't Hooft model [6]. It is assumed within this model that there is only a confining Coulomb-like linear interquark interaction. Then it manifestly provides spontaneous breaking of chiral symmetry in the vacuum via the self-energy loops and there appear Goldstone bosons [7, 8]. The complete spectrum of the $\bar{q}q$ mesons in the vacuum has been obtained in refs. [10, 11] and exhibits a fast chiral restoration with increasing J, for a review see ref. [12]. The meson spectrum at the finite chemical potential and zero temperature has recently been studied in [2]. It has been explicitly demonstrated that indeed at the chemical potential below the critical one the spectrum is similar to the one in the vacuum. Above the chiral restoration point the quarks are still confined and the physical spectrum consists of a complete set of exact chiral multiplets. In the present Letter we will essentially rely on the results of this paper. We will emphasize a crucially important point, that was not discussed at all: Above the chiral restoration point in the confined

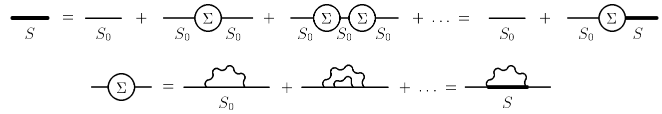

FIG. 3: Dressed quark Green function and the Schwinger-Dyson equation.

quarkyonic matter there necessarily appears a gap in the excitation spectrum. This property is explicitly seen in the context of this model, and is easily understandable. At the same time it must be a general property of the quarkyonic matter in the large N_c QCD above the chiral restoration point.

We work in the chiral limit and the two flavor version of the model is considered. The global chiral symmetry of the model is $U(2)_L \times U(2)_R$ because in the large N_c world the axial anomaly is absent. The model is described in great detail in references [11, 12] so here we review only those points that are important for understanding of the principal question of the present Letter. The only interquark interaction in our case is a linear instantaneous Lorentz-vector potential that has a Lorentz structure of the Coulomb potential:

$$K^{ab}_{\mu\nu}(\vec{x}-\vec{y}) = g_{\mu 0} g_{\nu 0} \delta^{ab} V(|\vec{x}-\vec{y}|); \qquad \frac{\lambda^a \lambda^a}{4} V(r) = \sigma r, \tag{1}$$

where a, b are color indices. Parameterising the self-energy operator in the form

$$\Sigma(\vec{p}) = A_p + (\vec{\gamma} \cdot \hat{\vec{p}})[B_p - p], \tag{2}$$

where functions A_p and B_p are yet to be found, the Schwinger-Dyson equation for the self-energy operator in the rainbow approximation,which is valid in the large N_c limit for the instantaneous interaction, see Fig. 3, is reduced to the nonlinear gap equation for the chiral (Bogoliubov) angle φ_p,

$$A_p \cos \varphi_p - B_p \sin \varphi_p = 0, \tag{3}$$

where

$$A_p = \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} V(\vec{p}-\vec{k}) \sin \varphi_k, \tag{4}$$

$$B_p = p + \frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \, (\hat{\vec{p}} \cdot \hat{\vec{k}}) V(\vec{p}-\vec{k}) \cos \varphi_k. \tag{5}$$

A key point is that the infrared regularization is required for the Coulomb gauge "gluon" propagator $\sim 1/q^4$. Then all observable color-singlet quantities, like the quark condensate or hadron mass must be independent on the infrared cut-off parameter in the infrared limit, i.e., when this parameter approaches 0. On the other hand, the colored quantities, like the single quark Dirac operator must be infrared divergent, which means that the single quark is removed from the spectrum and it is confined. This actually represents a necessary condition for confinement of quarks. It was demonstrated that indeed in the vacuum all these requirements are satisfied [7, 8, 9, 10, 11].

Once the gap equation is solved in the infrared limit one obtains a quark Green function. Given this quark Green function we are in a position to solve the Bethe-Salpeter equation. The homogeneous Bethe-Salpeter equation for the quark-antiquark bound states in the rest frame with the instantaneous interaction is given as

$$\begin{aligned}\chi(m,\vec{p}) &= -i\int\frac{d^4q}{(2\pi)^4}V(|\vec{p}-\vec{q}|)\,\gamma_0 S(q_0+m/2,\vec{p}-\vec{q})\\ &\times\ \chi(m,\vec{q})S(q_0-m/2,\vec{p}-\vec{q})\gamma_0. \end{aligned} \tag{6}$$

Here m is the meson mass and $\vec{p}$ is the relative momentum. The infrared divergence cancels exactly in this equation [11] and it can be solved either in the infrared limit or for very small values of the infrared regulator. Even though the single quark Dirac operator is divergent in the infrared limit, the color-singlet hadrons are well defined and finite-energy systems. The spectrum of mesons in the vacuum has been calculated in refs. [10, 11] and is reviewed in ref. [12]. Since the chiral symmetry is spontaneously broken there are four well defined Goldstone bosons with zero mass with quantum numbers $I = 1, 0^{-+}$ and $I = 0, 0^{-+}$. We remind that there are no vacuum fermion loops in this large N_c model and hence the $U(1)_A$ symmetry is broken only spontaneously.

Assume now that we have a finite chemical potential at zero temperature and hence all levels below the Fermi momentum p_f are occupied. Consider a "probe quark" that is brought into the system. In order to see the properties of this quark we have to solve the gap equation (3) - (5), but the integration starts not from $k = 0$, but from $k = p_f$, because all levels below the Fermi momentum are Pauli blocked. The points $p < p_f$ are irrelevant for this probe quark which is always on top of the Fermi sea. The gap equation for this probe quark has been solved at any finite chemical potential in ref. [2] and below we overview those results that are crucial for our present conclusions.

Once the Fermi momentum p_f is below the critical value $p_f < p_f^{cr}$, then there is always a nontrivial solution of the gap equation $\varphi_p \neq 0$ and the chiral symmetry gets dynamically broken that is manifested in the nonzero dynamical mass of quarks, $M(p)$, in the nonvanishing quark condensate, as well as in the presence of the massless Goldstone bosons. However, above the critical Fermi momentum, $p_f > p_f^{cr}$, there is no nontrivial solution to the gap equation and the only solution is trivial, $\varphi_p = 0$, with identically vanishing dynamical mass and quark condensate. Hence the chiral symmetry gets restored at $p_f = p_f^{cr}$ and one obtains the chiral restoration phase transition at the corresponding chemical potential. Above the chiral restoration point the Lorentz scalar self-energy of quarks identically vanishes, $A_p = 0$. However, the Lorentz spatial-vector part $B_p - p$ does not vanish and is in fact the infrared divergent quantity. Hence the Dirac operator for this single quark is still infrared divergent. This implies that even in the chirally restored phase the single quark *above the Fermi sea* is confined and cannot be observed. Said differently, this means that there are no single quark excitations of the quarkyonic matter even in the chirally restored phase. This property crucially distinguishes the quarkyonic matter from all usual Fermi systems.

It does not mean, however, that there are no excitations above the quarkyonic Fermi sea. A color-singlet hadron constructed out of such probe quarks is a finite and well defined quantity. The single quark infrared divergence cancels exactly in the color singlet hadron and such a hadron is a finite and well defined quantity even in the chirally restored phase. Of course, all momenta of quarks inside such a hadron are above the Fermi momentum. Hence one *can excite the system, but the excitation will consist not of the massless single quarks, but rather of the massive color singlet hadrons*. Hadrons with the lowest energy in this case have the quantum numbers of the $(1/2, 1/2)_a$ and $(1/2, 1/2)_b$ chiral multiplets with $J = 0$. In the symmetry broken world these multiplets contain the Goldstone bosons and the quark condensate.

Above the chiral restoration point their mass is *not zero*, see Fig. 4. *Then there is a gap in the excitation spectrum of the quarkyonic matter. This gap can be arbitrary large depending on the chemical potential.* It is a principal point of the present Letter. A mechanism for this gap is quite unusual. It arises not because of the condensation of the Cooper-like pairs in the system. There is no such a condensation in our case. It arises exclusively due to the fact that above the chiral restoration point the lowest excitation is a confined but chirally symmetric hadron with the non zero mass.

Below the critical Fermi momentum the system is confined and the chiral symmetry is broken. There are gapless excitations in the system: One can always create an arbitrary amount of Goldstone bosons. Above the chiral restoration point the system is still confined, but the chiral symmetry is restored. There are neither Goldstone excitations nor the single quark excitations in this case. Then there appears a gap in the excitation spectrum. This gap will essentially determine properties of the quarkyonic matter above the chiral restoration point. In particular, a presence of the gap implies some processes in the system that proceed without dissipation of energy. As it is well known, a presence of a gap is crucial for formation of the superfluid and superconducting

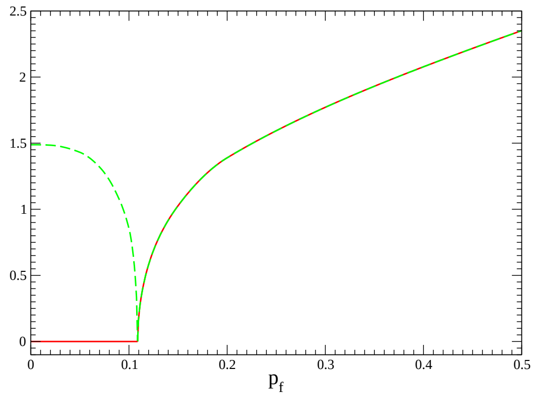

FIG. 4: Masses of the pseudoscalar 0^{-+} (solid) and scalar 0^{++} (dashed) mesons in units of $\sqrt{\sigma}$ as functions of the Fermi momentum in the same units.The chiral restoration point is at $p_f^{cr} = 0.109$.

systems. In our case the gap happens, however, without condensation of the fermionic system into a quasibosonic system. Hence one should expect that some properties of the quarkyonic matter above the chiral restoration point should be different as compared to the standard quasibosonic superfluids and superconductors.

Within the present simplified model we considered a step function momentum distribution near the Fermi surface, like in the Fermi gas. In reality, however, there are no reasons to expect such a simple Fermi distribution function and there should be some smoother distribution function near the Fermi surface even at zero temperature. However, such a fine detail of the Fermi distribution is unimportant for our main conclusions. Indeed, consider a probe quark well above the Fermi surface. For this quark all details of the momentum distribution near the Fermi surface are not important and all qualitative outputs of the gap and the Bethe-Salpeter equations with the step-function distribution will be valid.

The other critical question is to which extent our main conclusion depends on our specific model. Actually for a presence of the gap in the quarkyonic matter one needs only some very general ingredients. The system must be confined and it certainly follows from the large N_c arguments for pressure. At the same time the chiral symmetry must be restored at some critical chemical potential. Then our conclusions are quite general and do not rely specifically on the model. This model suggests an insight and a possible microscopic scenario.

3. Conclusions. As a conclusion we have demonstrated that above the chiral restoration point the quarkyonic matter represents a Fermi system with a gap. This gap increases with the chemical potential and is arbitrary large. Then there should be some processes without dissipation of energy, like in the superfluid or superconducting systems. A mechanism of the gap formation in the quarkyonic matter is essentially different as compared to the standard fermionic systems. In the latter it proceeds via the condensation of the quasibosonic Cooper pairs. In our case there is no such a condensation and the mechanism is based on the simultaneous properties of confinement and manifest chiral symmetry of hadrons on top of the Fermi sea above the critical density.

We have demonstrated this property in the case of the large N_c world. Then such an unusual fermionic system exists at least in the large N_c limit and consequently represents a scientifically well defined system and it is a valid and well formulated question to study properties of such a Fermi system. How will the properties of the quarkyonic matter differ at $N_c = 3$? At present we cannot answer this question. But it is difficult to imagine that there could be a dramatic difference in this respect between the real world and the large N_c world. Indeed, properties of QCD that are crucial for the present issue are confinement, chiral symmetry and its spontaneous breaking. They are known to persist both in the large N_c world and with $N_c = 3$. Then there is a reason to believe that what we have demonstrated for the large N_c world will still be valid in the real world.

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